MAR 13 1960

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N-82114

[redacted]

N65-82313	
(ACCESSION NUMBER)	(THRU)
17	None
(PAGES)	(CODE)
CR 60964	
(NASA CR OR TMX OR AD NUMBER)	(CATEGORY)

FACILITY FORM 602

JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA 3, CALIFORNIA

National Aeronautics and Space Administration
Contract No. NASw-6

Technical Release No. 34-16

THE JET PROPULSION LABORATORY LUNAR-PROBE TRACKING AND ORBIT-DETERMINATION PROGRAM

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National Aeronautics and Space Administration
Operated by
California Institute of Technology
Pasadena, California
March 10, 1960

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ABSTRACT

During the past two years, the Jet Propulsion Laboratory has developed, in conjunction with its lunar probe operations, a real-time tracking computation program. Designed for the IBM 704, this program accepts coded tracking data, computes a best-fit orbit, and predicts pointing information for the tracking stations.

In its present form, this program can handle lunar-probe orbits with precision sufficient for most scientific requirements. However, certain modifications are required to adapt it for planetary probes.

The purpose of the present paper is to describe some of the details of the computer program. Particular attention is given to the problem areas, and the reasons for using particular techniques.

In addition, there is a short description of the results of using this program during the Pioneer IV operation, and subsequently to analyze the Pioneer IV data.

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[1]This paper presents results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under Contract No. NASw-6, sponsored by the National Aeronautics and Space Administration.

I. INTRODUCTION

The computation program used at the Jet Propulsion Laboratory for real-time orbit determination is the outcome of about two years' work in conjunction with lunar probe activity. Orginally designed for application to lunar probes, the program has been found valuable in certain satellite applications, and is expected to apply with only minor changes to interplanetary probes. It can handle many types of data, and can be used efficiently either in real time to produce antenna-pointing information, or in postflight analysis. Thus, it is considered to be a versatile general-purpose program, which can be expanded to keep up with this fast-changing field.

Perhaps the simplest way of looking at orbit determination is as a problem in curve-fitting. We are presented with a set of data points obtained from various observation sites, and we are required to fit a trajectory-curve to these points. In concept, the solution to this problem is simple: a least-squares technique is used to find the trajectory which minimizes the sums of the squares of the errors of observations. This is a standard statistical procedure which poses no problem in itself.

However, problems do arise when we begin to delve into details. The curve we are fitting, i.e., the trajectory, is complicated; because of the required precision, it can be represented only by its differential equations, not by an explicit closed form. The data from the observation stations are noisy. Often there are biases whose nature is obscure. The noise itself may be far from Gaussian. In addition, there are many other sources of difficulty.

The remainder of the paper will be devoted to a discussion of some of these problems and the method used to solve them.

II. DESCRIPTION OF THE COMPUTATION PROGRAM

As stated above, the computation of the orbit is based on a least-squares fit and is programmed for IBM 704 and 7090.[2] Six trajectory parameters, namely the three components of velocity and the three components of position at the injection time, are determined in such a manner that the sum of the squares of the observation errors is minimized (Ref. 1).

To start the computation, it is necessary to have a reasonable estimate of the values of the trajectory parameters, as obtained (for example) from the so-called nominal trajectory (or the preflight standard). Let these be denoted by the vector

$$X = (X^1, \ldots, X^6)$$

whose first three components represent the injection position, the second three the injection velocity. Then, the sum of the squares of the observation errors is written in terms of increments in the X^i, thus

$$S = S(\Delta X^1, \ldots, \Delta X^6)$$

Setting the six partial derivatives of S each equal to zero gives six equations in the six unknowns ΔX^i:

$$\frac{\delta S}{\delta(\Delta X^i)} = 0 \qquad i = 1, 2, \ldots, 6$$

[2]The program will be modified slightly for use with the 7090 computer when it is installed.

If the determinant

$$\left| \frac{\delta^2 S}{\delta(\Delta X^i)\ \delta(\Delta X^j)} \right|$$

does not vanish, then the six equations determine a set of values for the ΔX^i which when added to the X^i gives an estimate of the parameters belonging to the best-fit trajectory.

Since this process can be carried through only after simplifying approximations (e.g., linearizing the equations) have been made, the derived values of X^i are only approximate. However, if the data is adequate, and the simplifying assumptions reasonable, the new value of X^i should give a closer approximation to the actual trajectory than the original estimate gives. Repeating the process should give a still closer approximation.

Thus, we have set up an iteration procedure which--we have found by experience--converges under reasonable conditions. The mathematical question of determining necessary and sufficient conditions for convergence is still unanswered. However, as a practical matter this question may be considered academic.

In putting the above computation procedure into practice, it is necessary to perform various auxiliary computations, each of which is a major programming effort in its own right. The trajectory itself--the coordinates of position and velocity at various instants of time--must be computed for each of the iterations described above. We use a Cowell integration of the equations of motion in rectangular co-ordinates, based on a fourth-order Runge-Kutta method.

The effects of Earth, Moon, Sun, and each of the major planets are included in the force field. The Earth's oblateness is accounted for by including the J and D terms in its gravity potential. The other celestial bodies are represented as point masses.

At least once during each orbit determination, we must compute the sensitivity coefficients, that is, the partial derivatives of each observation with respect to each trajectory parameter X^i. This is accomplished by integrating the differential equations of these partials along with the trajectory itself. In practice, this amounts to integrating a set of thirty-six linear equations simultaneously with the integration of the basic trajectory equations.

Another major portion of the program is the handling of the ephemeris information. It is essential in the computation to have position data for Earth, Moon, Sun, and those planets being used in the computation. Ephemeris data are stored in the memory, using information obtained from the Nautical Almanac office of the U.S. Naval Observatory (Ref. 2).

There are many other auxiliary computations, including coordinate conversions, computation of various statistical quantities, corrections for refraction and aberration and estimate of biases in the data.

To summarize, the orbit-determination program is a least-square curve-fitting procedure characterized by

1. The parameters for the least-square fit are the six coordinates of the probe at injection.

2. The quantity to be minimized for the least-square process is the sum of the squares of the errors of observation, i.e., the errors in the raw data.
3. Ephemeris information is stored in memory in terms of position data of each celestial object of concern.
4. The probe trajectory is obtained by integrating the equations of motion in rectangular coordinates.
5. Sensitivity coefficients are computed directly from their differential equations.

III. PROBLEM AREAS

A. Geometrical Indeterminacy

In general, six data points are required to determine the six injection parameters. For example, two angles and range from one observing station is three data points; taken at two times, this gives six. There is only one trajectory that fits these six data points simultaneously. Given these six points, our orbit determination procedure computes the appropriate injection coordinates.

It may happen, however, because of the geometry of the station or the type of data points or both, that the orbit is not uniquely determined by the data--even when many more than six data points are available. As a simple example, suppose that there are three stations, each observing two angles and range, and they each get data points simultaneously at a particular instant. This yields nine data points altogether. However, each station's information is equivalent to probe position: we have redundant determination of position, but no determination of velocity. How does this situation show up in the orbit-determination program?

The critical step, mathematically, in solving for the injection conditions is the associated matrix inversion. Without going into details, we merely note that the elements of the matrix in question consist of sums of products of the sensitivity factors for each data type. When the data is insufficient for uniquely determining the trajectory, this matrix is ill-conditioned.

It is always desirable to set up observation sites and to choose observables so as to avoid geometric indeterminacy, i.e., to ensure that the sensitivity matrix is not singular, and in fact is as far from singular as possible. However, it is not always possible to obtain the type of data needed: the observation sites may be all in a line, or angle data may be the only information available.

In such instances, we are forced to use an abridged computation, one in which (for example) we assume a priori knowledge of the position at injection, and solve only for the velocity. This procedure is particularly valuable during the first hour or so of a flight operation when the number of data points is small and the probe has not been acquired by all the observation sites.

However, we have attempted to minimize the chance of indeterminacy by taking precautions within the computation procedure. We have adopted two devices, each of which is intended to improve the conditioning of the sensitivity matrix. The first is to include in each diagonal element a contribution from the a priori knowledge of the injection coordinates. Since we know, for example, that the injection altitude must be above the surface of the Earth, we can get a conservative estimate of the altitude standard deviation, and include this in the appropriate diagonal element of the sensitivity matrix.

The second is to compute the sensitivity coefficients directly from the differential equations, instead of by differences. The net result is to get these coefficients accurate to about six significant digits instead of only three--with a corresponding improvement in the conditioning of the matrix.

B. Noisy Data

The Laboratory's experience has been primarily with radar angle data and doppler frequency data. The doppler data have been very smooth, but, unfortunately, badly biased. The angle data have been reasonably smooth and we have been able to calibrate the major portion of the biases. The principal problem with noisy data is how to weight it properly.

This orbit-determination program has the capability of two types of weighting. One, within a particular data type, assigns a weight to each data point according to some geometrical criterion, for example elevation angle above the horizon. This procedure assumes a priori knowledge of the degradation of signal with respect to some parameter.

The second weighting is designed for internal determination, that is, it is a function of the data and is determined by the computation program. It assigns relative weight to each data type. The basis of these weights is the smoothness of the data: the smoother the data, the higher the weight. This scheme must be used with care, however, because it may happen that a very smooth data type has a large uncalibrated bias. The result will be to fit the smooth, biased data, and to discard the remaining data.

C. Biases

Every observation instrument is contaminated with bias of one sort or another, that is, with consistent error. Since the error is consistent, it is evident that a calibration procedure should be able to detect the magnitude of the bias, which can then be used to adjust subsequent data. The instruments used in tracking for orbit determination are so calibrated, and the appropriate corrections included in the computation. However, because of the extreme precision required of these instruments, because of the difference between the conditions during tracking a probe and those that can be set up for calibration, and because of drift in the biases, there is usually a small but significant uncorrected bias remaining in the system.

To cope with this situation, the computation program has been designed to do some of the calibration internally, during a tracking operation. As presently set up, the program can detect a constant bias in the observations at one station, provided the data is determinate. The procedure amounts to augmenting the number of injection parameters by including the biases, thus making a total of as many as nine parameters to be solved for. The corresponding sensitivity matrix is of ninth order.

Of course, this procedure is appropriate only for constant bias. When the bias drifts, or is a function of range or elevation, other corrective measures are needed.

IV. APPLICATION TO PIONEER IV

In March 1959, the National Aeronautics and Space Administration employed the Juno II rocket system to place a scientific payload on an escape trajectory in the vicinity of the Moon. This probe passed the Moon at a distance of approximately 60,000 km, and continued on into its own orbit about the Sun. The JPL orbit-determination program was used to assimilate the tracking data, predict the orbit, and produce pointing information for various tracking sites.

The tracking sites were located at the launch site (Cape Canaveral, Florida), Puerto Rico, and Goldstone, as shown in Fig. 1.[3] The data types, estimated accuracies, and approximate visibility periods are listed below.

Station	Data Type	Estimated Standard Deviations	First-Pass Visibility Period
Launch	Doppler Frequency	10 m/sec	0 to 10 min.
Puerto Rico	Azimuth Elevation	0.2 deg 0.2 deg	6 min. to 13 1/2 hours
Goldstone	Hour Angle Declination	0.01 - 0.02 deg 0.01 - 0.02 deg	6 1/2 hours to 15 1/2 hours

Data points were fed by teletype from the observation sites to the computing center of JPL at a maximum rate of a set of points every 10 sec.

The first 15 minutes of Cape data after injection were used to make pointing predictions for Puerto Rico for a time one hour later than the last data point used. These predictions were subsequently found to agree with the

3 The Figure also shows the Jodrell Bank radio telescope, which independently tracked the probe.

Puerto Rico observations to within less than 0.2 deg. The corresponding injection parameters differ from the present best estimates by 12 km in injection altitude and 30 meters/sec in velocity.

With 3 1/2 hours of data from Puerto Rico, the acquisition prediction for Goldstone was found to agree with observations to within 0.1 degree. The corresponding initial conditions differ from the best estimate by 2 km in altitude, 0.05 deg in latitude and longitude, 5 meters/sec in velocity, and 0.1 deg in the velocity angles. At the distance of the Moon, the accuracy of the probe position as determined by the complete data is estimated to be 100 km.

This brief resume of the Pioneer IV results is presented merely to show that the computing program is an effective tool for real time orbit predictions. It has also been used to good advantage in post-flight orbit analysis.

V. CONCLUSION

An orbit-determination program has been described which can handle lunar probes and the early phase of planetary probes with the precision demanded by present scientific requirements. It can fit satellite orbits, too, provided the data do not encompass too many periods of the satellite. The limitation here is one of machine time, as computation takes approximately two minutes per period.

The most urgent areas in which the program needs improvement are (a) inclusion of a good method of compressing data, so as to reduce computer time, (b) inclusion of automatic method of reducing the order of the sensitivity matrix when it is ill-conditioned, and (c) inclusion of corrections in the observations to account for the finite velocity of light.

These items are all either under study, or in process of being programmed.

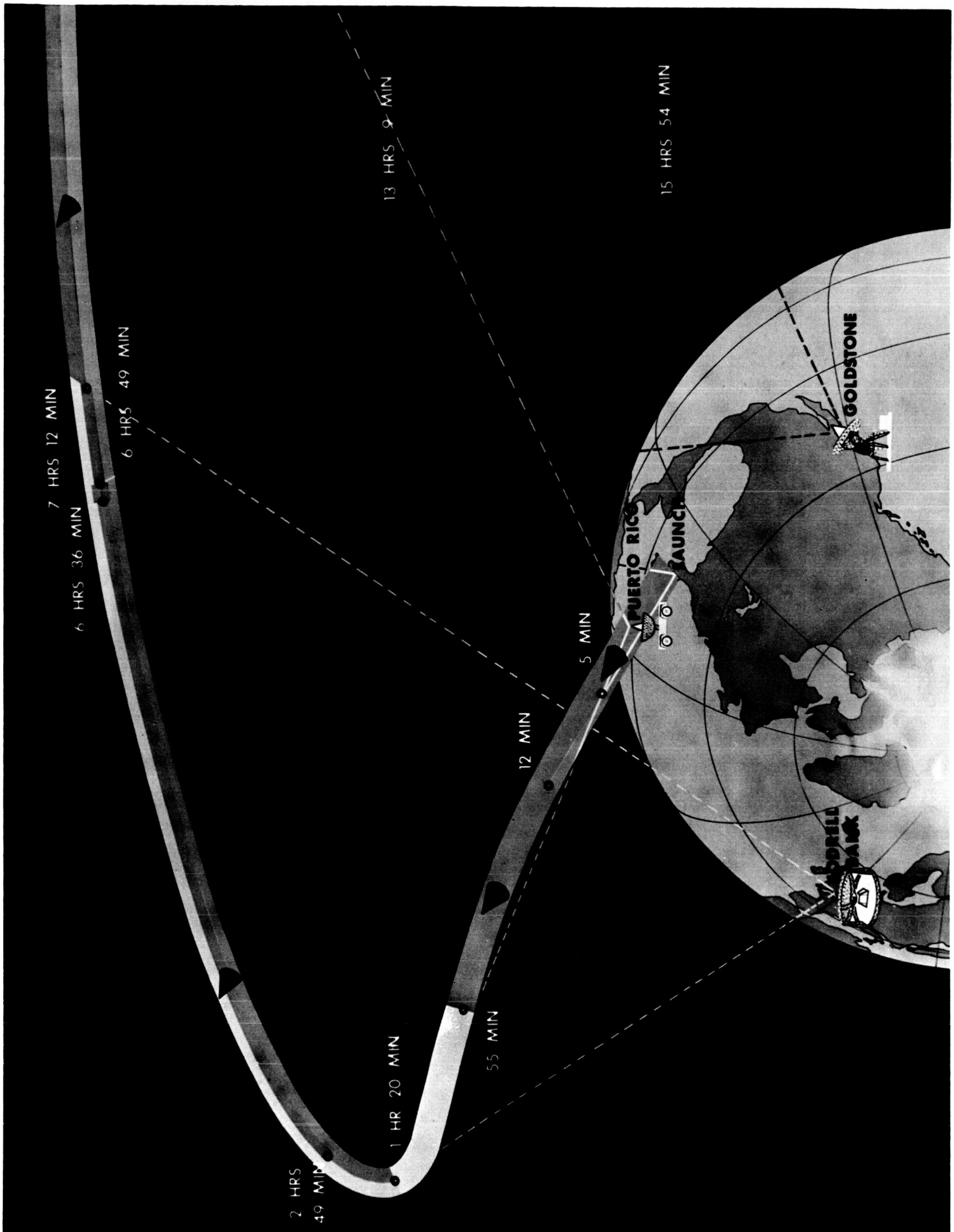

Fig. 1. Initial tracking coverage of Pioneer IV

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